Exercise 1.1: McCulloch-Pitts Neural Net (5 Points)
Give for each of the Boolean functions a McCulloch-Pitts ANN that solves it and is as small as possible
(\lor = \text{or}, \land = \text{and}, \overline{x} = \text{negation}).

a) \[ f(x) = \overline{x}_1 \overline{x}_2 \overline{x}_3 \lor x_1x_2 \lor x_2x_3 \]
b) \[ g(x) = x_1x_2x_3 \lor x_1x_2\overline{x}_4 \lor x_1x_4 \land \overline{x}_2. \]

Exercise 1.2: Perceptron Classifier (5 Points)
A class \( C \subset \mathbb{R}^2 \) contains all points in the quadrangle with the vertices \( \vec{a} = (1,1)^T, \vec{b} = (2,-2)^T, \vec{c} = (0,-1)^T, \) and \( \vec{d} = (-1,1)^T. \) Manually construct an ANN with perceptrons that can decide whether an input vector \( (x,y)^T \) is inside the quadrangle or not. Give the net and describe the construction, especially the calculation of the weight vectors.

Exercise 1.3: Perceptron Learning (6 Points)
Implement the perceptron learning algorithm (lecture 1, slide 27) in the programming language R. Apply the algorithm to the given test sets (set1.txt, set2.txt, set3.txt). The files contain one example per line and the classification as last value (1: positive example, 0: negative example) of each line. Implement a stopping criterion so that the algorithm cannot run infinitely. In each iteration during training, calculate the measures

\[ \text{recall} = \frac{tp}{tp + fn} \quad \text{and} \quad \text{precision} = \frac{tp}{tp + fp}. \]

\[
\begin{bmatrix}
  tp &= \text{number of correct (true) positive classifications} \\
  fp &= \text{number of incorrect (false) positive classifications} \\
  tn &= \text{number of correct (true) negative classifications} \\
  fn &= \text{number of incorrect (false) negative classifications}
\end{bmatrix}
\]

Finally, visualize the result by plotting the two measures against each other. Interpret the figure.

Exercise 1.4: Classification into more than two classes (4 Points)
How could an ANN be enhanced such that it is able to classify inputs into more than two classes? Think about at least three concepts and describe in detail.

Recommended reading
Read the Wikipedia article about the so-called “AI Winter” [https://en.wikipedia.org/wiki/AI_winter] especially the sections “The abandonment of connectionism in 1969” and “Lasting effects of the AI winters.”