Exercise 5.1: Fuzzy Sets (5 Points)

Give membership functions that define the following fuzzy sets appropriately from your point of view:

(a) $A_1$: Young people, $B_1$: middle-aged people, $C_1$: old people.
(b) $A_2$: Slow car, $B_2$: fast car, $C_2$: too fast car.
(c) $A_3$: Great weather, $B_3$: Nice weather, $C_3$: bad weather.

To do this, first define a crisp set (domain, unit) over which the fuzzy sets are defined, respectively (e.g. the crisp set $X = \{x \in \mathbb{R} | x \in [0, 100]\}$ would be appropriate for fuzzy sets 'cold coffee; hot coffee' and the unit is temperature in °C). Give the membership functions for the triples of fuzzy sets as a (drawn/plotted) graphic with the appropriate domain on the horizontal axis and the degree of membership on the vertical axis, and as a formula.

Perform the operations $A_{ci}, B_{ci}, A_i \cup B_i, A_i \cap B_i$ for all $i = \{1, 2, 3\}$ and give the resulting sets by drawing/plotting their membership function, respectively.

Exercise 5.2: $\alpha$-cuts (5 Points)

Let $A$ and $B$ be fuzzy sets over the crisp set $X$.

The $\alpha$-cut $A^{\geq \alpha}$ of $A$ is defined as $A^{\geq \alpha} := \{x \in X | A(x) \geq \alpha\}$.

Prove that the following statements are correct:

(a) $A \subseteq B \iff \forall \alpha \in [0, 1) : A^{\geq \alpha} \subseteq B^{\geq \alpha}$

(b) $A = B \iff \forall \alpha \in [0, 1) : A^{\geq \alpha} = B^{\geq \alpha}$