Exercise 7.1: Fuzzy Implication (5 Points)

a) Use the increasing generator \( g(x) = \sqrt{x} \) to derive a fuzzy implication. Does the resulting implication fulfill the axiom of contraposition?

b) Check for all fuzzy implications below if they fulfill the axiom of contraposition:

- Reichenbach \( \text{Imp}(a, b) = 1 - a + a b \)
- Lukaciewicz \( \text{Imp}(a, b) = \min\{1, 1 - a + b\} \)
- Goguen \( \text{Imp}(a, b) = 1_{[a \leq b]} + b \cdot 1_{[a > b]} \)

Exercise 7.2: Fuzzy Inference (5 Points)
Consider the membership functions for the linguistic terms of the linguistic variable mark. Notice that outside the given range their values are zero!

\[
\begin{align*}
\text{very_good}(x) &= \exp(-2x^2) \text{ for } x \geq 1 \\
\text{good}(x) &= -(x - 1)(x - 3) \text{ for } x \in (1, 3) \\
\text{fair}(x) &= -(x - 2)(x - 4) \text{ for } x \in (2, 4) \\
\text{bad}(x) &= \min\{1, \frac{1}{2}(x - 3)\} \text{ for } x > 3
\end{align*}
\]
Below you can find the membership functions for the linguistic terms of the linguistic variable `learning_time`. Again, outside the given range their values are zero!

\[
\begin{align*}
\text{huge}(x) &= \min\{x - 5, 1\} \text{ for } x \geq 5 \\
\text{big}(x) &= -\frac{4}{9}(x - 3)(x - 6) \text{ for } x \in (3, 6) \\
\text{low}(x) &= \min\{2 - x, 1\} \text{ for } x < 2
\end{align*}
\]

Based on the fuzzy proposition

if `learning_time` is big then `mark` is good,

the Lukaciewicz implication and the max-prod composition deduce the resulting fuzzy set over learning time for the given fuzzy fact

`mark` is fair.

Sketch the membership function.