Exercise 4.1: Fuzzy Inference (5 Points)
Consider the membership functions for the linguistic terms of the linguistic variable \textit{mark}. Notice that outside the given range their values are zero!

\begin{align*}
\text{very\_good}(x) &= \exp(-2x^2) \text{ for } x \geq 1 \\
\text{good}(x) &= -(x-1)(x-3) \text{ for } x \in (1,3) \\
\text{fair}(x) &= -(x-2)(x-4) \text{ for } x \in (2,4) \\
\text{bad}(x) &= \min\{1, \frac{1}{2}(x-3)\} \text{ for } x > 3
\end{align*}
Below you can find the membership functions for the linguistic terms of the linguistic variable \texttt{learning time}. Again, outside the given range their values are zero!

\[
\begin{align*}
\text{huge}(x) &= \min\{x - 5, 1\} \text{ for } x \geq 5 \\
\text{big}(x) &= \frac{4}{9}(x - 3)(x - 6) \text{ for } x \in (3, 6) \\
\text{low}(x) &= \min\{2 - x, 1\} \text{ for } x < 2
\end{align*}
\]

Based on the fuzzy proposition

\[
\text{if learning time is big then mark is good,}
\]

the Lukaciewicz implication \( \text{Imp}(a, b) = \min\{1, 1 - a + b\} \) and the max-prod composition deduce the resulting fuzzy set over learning time for the given fuzzy fact

\[
\text{mark is fair.}
\]

Sketch the membership function. Hint: Discretize the function and use a table of values.

\textbf{Exercise 4.2: Fuzzy Implication (5 Points)}

a) Use the increasing generator \( g(x) = \sqrt{x} \) to derive a fuzzy implication. Does the resulting implication fulfill the axiom of contraposition?

b) Check for all fuzzy implications below if they fulfill the axiom of contraposition:

- Reichenbach \( \text{Imp}(a, b) = 1 - a + a b \)
- Lukaciewicz \( \text{Imp}(a, b) = \min\{1, 1 - a + b\} \)
- Gödel \( \text{Imp}(a, b) = \begin{cases} 1 & a \leq b \\ b & \text{otherwise} \end{cases} \)
Exercise 4.3: Fuzzy Controller for a vehicle (10 Points)

Implement in R a Mamdani controller for a car. The car shall drive speedily towards a wall and come to a standstill as close as possible to the wall without crashing into it.

The car gives the measured values of its current speed and its current distance to the wall. The actuating variable is the thrust that accelerates the car forwards or backwards. The driving of the car shall be simulated in discrete time steps.

a) Model the membership functions for the linguistic variables *speed*, *distance to wall*, and *thrust*.

b) Setup an appropriate fuzzy rule system for a Mamdani controller.

c) Implement the control loop for the Mamdani controller using the center of gravity method for defuzzification.

d) Give a short documentation of your source code.

e) Start the car in big distance to the wall with low, middle and high initial speed. Report for each initial speed a successful run of the car. Do this by plotting distance, speed and thrust over time.