On the Hybridization of SMS-EMOA and Local Search for Continuous Multiobjective Optimization

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ABSTRACT

In the recent past, hybrid metaheuristics became famous as successful optimization methods. The motivation for the hybridization is a notion of combining the best of two worlds: evolutionary black box optimization and local search. Successful hybridizations in large combinatorial solution spaces motivate to transfer the idea of combining the two worlds to continuous domains as well. The question arises: Can local search also improve the convergence to the Pareto front in continuous multiobjective solutions spaces? We introduce a relay and a concurrent hybridization of the successful multiobjective optimizer SMS-EMOA and local optimization methods like Hooke & Jeeves and the Newton method. The concurrent approach is based on a parameterized probability function to control the local search. Experimental analyses on academic test functions show increased convergence speed as well as improved accuracy of the solution set of the new hybridizations.

Categories and Subject Descriptors

G.1.6 [**Optimization**]: Global optimization; G.4 [**Math. Software**]: Algorithm design and analysis

General Terms

Algorithms

Keywords

Hybrid Evolutionary Multiobjective Algorithm, Local Search, Memetic Algorithms, Hybrid Metaheuristics

1. INTRODUCTION

The optimization of multiple conflictive objectives is a hard problem. Effective stochastic optimization methods have been developed to solve multiobjective problems. In particular the S-Metric Selection Evolutionary Multiobjective Algorithm, in short SMS-EMOA [1] – turned out to be one of the most successful evolutionary multiobjective optimizers. In the last years a new and successful trend appeared: hybridizing evolutionary algorithms with local search. When usual stochastic operators cannot contribute to an improvement, e.g. because of low success rates, more specialized local search methods may accelerate the search. Up to now a notable success can be observed in large combinatorial search spaces, e.g. [10, 12, 13, 11, 5, 14]. In general, a hybrid metaheuristic is a metaheuristic combined with a search technique that is often a classical optimizer, e.g. gradient descent or the Newton method. The question arises: Can the hybridization with local search improve the optimization process in continuous multiobjective solution spaces? Or more precisely: Can local search techniques like Steepest Descent, Hooke & Jeeves or Newton speed up the search in combination with the well-known and successful SMS-EMOA? To answer this question, we integrated local search into the SMS-EMOA and performed a detailed experimental analysis.

The invoked methods and related research on hybrid multiobjective optimization are introduced in the next section. Section 3 presents the proposed relay hybrid of SMS-EMOA and local search and Section 4 the concurrent hybrid. The experimental studies are integrated in the sections respectively. We conclude with a summary and topics of future research in Section 5.

2. MULTIOBJECTIVE HYBRIDIZATION

In the following, we introduce the foundations of hybrid metaheuristics for multiobjective optimization techniques, in particular the invoked methods of our work.

2.1 Optimization with SMS-EMOA

A multiobjective optimization problem consists of m objective functions $f_1(x), f_2(x), ..., f_m(x)$, with $f_i(x) : \mathbb{R}^n \rightarrow \mathbb{R}$, $\forall i \in 1, ..., m$ to be minimized.

The SMS-EMOA [1] belongs to the indicator-based EMOAs that optimize a set of solutions according to a certain scalar quality measure—the S-metric in this case. The S-metric or hypervolume indicator measures the size of the space dominated by the population. It is integrated into the selection operator of SMS-EMOA which aims for maximization of the S-metric and thereby guides the population towards the Pareto front. A $(\mu+1)$ (or steady-state) selection scheme is applied: In each generation discarding the individual that contributes least to the S-metric value of the population. The invoked variation operators are not specific for the SMS-EMOA but taken from literature, namely polynomial mutation and simulated binary crossover with the same parametrization as for NSGA-II [4].

2.2 Local Search in Continuous Solution Spaces

Local search techniques are classic hill-climbing methods. They start at a certain point in the search space and move iteratively in a descent direction until they reach a local optimum of the function. For a comprehensive introduction to local search techniques we refer the interested reader to Schwefel [18] and Box *et al.* [2]. Basically, local search methods for numerical optimization can be partitioned into the following three classes:

- Derivative-free methods, e.g. Direct Search methods
- First-order methods, e.g. Steepest Descent
- Second-order methods, e.g. Newton's method

Usually, local search methods are not designed to solve multiobjective problems. Direct Search methods for instance compare objective function values to find a local minimum. But in the multiobjective case the optimizer must cope with an m-dimensional objective vector instead of a scalar value. Hence, it is no easy undertaking to use a single-objective local search method in multiobjective solution spaces. A possible answer to this problem is the scalarization of the objective function by minimizing the distance to an "utopian" point as described by Steuer[1] [21]. Another technique is to use the multiobjective Steepest Descent method by Fliege and Svaiter [8] or the Newton method by Fliege *et al.* [7]. These methods use first or second order derivatives of the objective function to find a descent direction of the function. Sections 2.2.1, 2.2.2 and 2.2.3 provide concrete instances of the three local search classes for the multiobjective case.

2.2.1 Hooke & Jeeves

The method of Hooke & Jeeves [9] belongs to the class of Direct Search methods. In particular, no additional derivative information is necessary. The fundamental advantage of Hooke & Jeeves is its ability to approximate the actual gradient direction. In each iteration an exploratory move along the coordinate axes is performed. Afterwards, the vectors of the last exploratory moves are combined to a projected direction that can accelerate the descent of the search vector. When the exploratory moves lead to no improvement in any coordinate direction, step sizes are reduced by a factor η – we will set $\eta = 0.5$. The search terminates after a number of predefined function evaluations or, alternatively, when the step size falls below a constant value $\varepsilon > 0$.

[1]The "utopian" point is a solution that strictly dominates all solutions.

The application of Hooke & Jeeves to multiobjective solution spaces is not possible without modifications. The algorithm makes use of comparisons of scalar function values, so that the quality measures of a multiobjective solution must be mapped to a scalar value first. This can be done by using the weighted Tchebycheff method as proposed by Steuer [21]. The algorithm minimizes the weighted Tchebycheff distance from the current position to an "utopian" point. Here, we deploy the weighted sum approach developed in [3].

2.2.2 Steepest Descent

Fliege and Svaiter [8] generalize the method of Steepest Descent for multiobjective optimization. The algorithm computes the Jacobian $J_f = (\nabla f_1, \ldots, \nabla f_m)$ of the objective function and tests if the current search vector $\vec{x}_k \in \mathbb{R}^n$ is locally Pareto-optimal. If the vector is locally Pareto-optimal, that is when no descent direction is available in the surrounding area, the algorithm stops. Otherwise, the algorithm computes a search direction $\vec{v}$ by using the gradient information from the Jacobian at point $\vec{x}_k$ as described by Fliege and Svaiter [8]:

$$\min_{v} \left(\max_{i=1,..,m} (\nabla f_i(\vec{x}_k)^T \vec{v}) + \frac{1}{2}||\vec{v}||^2 \right). \qquad (1)$$

Fliege and Svaiter show that the algorithm converges to a Pareto-optimal point without the need of a scalarization of the objective function. For details of the multiobjective Steepest Descent algorithm we refer to [8].

2.2.3 Multiobjective Newton Method

The multiobjective Newton method by Fliege *et al.* [7] is an extension of Newton's method for the single-objective case. The technique makes use of the gradients and Hessian matrices of all objectives to converge faster to an optimal solution. With the additional information of Hessians the search direction $\vec{v}$ can better be estimated by solving the following auxiliary problem as described by Fliege *et al.* [7]:

$$\min_{v} \left(\max_{j=1,..,m} (\nabla f_j(\vec{x}_k)^T \vec{v}) + \frac{1}{2}\vec{v}^T \nabla^2 f_j(\vec{x}_k)\vec{v} \right). \qquad (2)$$

The convergence rate of the algorithm is quadratic if the second derivatives are Lipschitz continuous. Since the multiobjective Newton method converges one order faster than multiobjective Steepest Descent it is potentially an excellent local search method for hybridization.

2.3 Types of Hybridization

The most important design decision for hybrid techniques concerns the way of information interchange between its components. In which order shall the components work together, which information is shared, and when? Can general hybridization rules be derived from theory or experiments? Talbi [23] and Raidl [16] tried to answer these questions and introduced taxonomies of hybridization techniques. They introduced the terms *relay* or *sequential*, and *coevolutionary* or *interleaved* hybrids. A relay hybrid is a simple successive execution of two or more algorithmic components. A stochastic method might pre-optimize coarsely while the local search performs fine-tuning and approximation of local optima. The coevolutionary hybrid is a nested approach. Typically, a local search method is embedded into an evolutionary optimizer: In each iteration the local search optimizes the offspring solutions until a predefined termination condition is fulfilled. Information is passed alternately between the components in the concurrent approach.

2.4 Related Work

The first work on hybridization with the SMS-EMOA stems from Emmerich *et al.* [6] who proposed a relay hybrid version of the SMS-EMOA. The main idea of the hybrid is to guide the local search by calculating the gradient of the $\mathcal{S}$-Metric. Emmerich *et al.* report a linear convergence to the optimum. Deb and Goel [3] employed scalar local search on a weighted sum surrogate function. Other proposals of hybrid multiobjective evolutionary algorithms are concurrent or memetic approaches. E.g. Sindhya *et al.* [20] have examined a hybrid version of the NSGA-II [4] with an integrated gradient descent method as local optimizer. They use an augmented scalarization function to map the multiobjective solution to a single scalar value. Schuetze *et al.* [17] present a hillclimber with sidestep in an iterative local search procdedure that can be integrated in an arbitrary multiobjective optimizer. Martínez and Coello Coello [15] propose to hybridize the NSGA-II with classical Direct Search techniques. Shukla [19] has also hybridized the NSGA-II with two gradient methods using a perturbation technique as mutation operator.

3. RELAY SMS-EMOA HYBRID

First, we concentrate on relay hybridization of the SMS-EMOA and the Steepest Descent method. After the description of the algorithmic concept, an experimental analysis on typical multiobjective test functions follows.

3.1 Algorithmic Description

Our first relay hybrid is based on the SMS-EMOA and the Hooke & Jeeves method. The relay hybridization requires the definition of an adequate termination criterion for the first optimizer. In literature following stopping criteria are proposed for hybrid algorithms:

1. Stop after k function evaluations.
2. Stop, when quality $\mathcal{Q}$ is reached.
3. Stop, when improvement over k iterations is smaller than $\varepsilon > 0$.

Termination criterion 2 is not applicable if the function is unknown. It is also difficult to choose adequate parameter settings for termination criterion 3 as stagnation may occur during the optimization process. Therefore we resort to the first criterion in this study. In the experimental analysis, see next paragraph 3.2, we analyze the point in time for a strategy switch systematically. After the strategy switch the local search is started. The hybrid method terminates when the local search terminates.

3.2 Experimental Analysis

We perform a systematic analysis of *when* to switch to local search. Therefore we investigate the influence of the ratio φ between function calls of the SMS-EMOA and local search. A value $\varphi = 0.0$ means that 0 percent of the maximal function calls are spent with SMS-EMOA. Figure 1 shows the results of the SMS-EMOA-Steepest Descent-relay hybrid in ten runs of:

1. pure optimization with Steepest Descent ($\varphi = 0.0$),

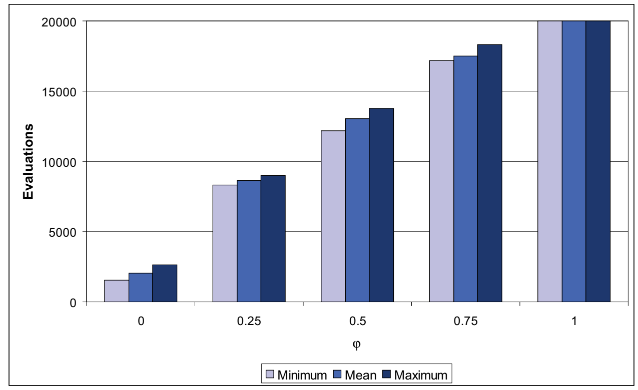

Figure 1: Function evaluations on ZDT1. The bars show different rates φ of total function evaluations between SMS-EMOA and Steepest Descent.

2. 25% of $20,000$ fitness function evaluations spent for SMS-EMOA and 75% spent for Steepest Descent[2] ($\varphi = 0.25$),
3. 50% of $20,000$ fitness evaluations for both SMS-EMOA and Steepest Descent ($\varphi = 0.5$),
4. as well as 75% of $20,000$ evaluations for SMS-EMOA and 25% for Steepest Descent ($\varphi = 0.75$),
5. and 100% for SMS-EMOA respectively ($\varphi = 1.0$).

As quality measure, we make use of the NDHV-measure (normalized-difference in hypervolume that has to be minimized) like Sindhya *et al.* [20]:

$$\text{NDHV} = (\text{HV}^* - \text{HV})/\text{HV}^*.$$

The value HV^* is the optimal hypervolume value for the problem, regarding to a fixed reference point R. In fact the optimal hypervolume value is usually not available for non-academic test functions, but can be computed in case of academic problems as the ZDT functions by Zitzler *et al.* [24], so that NDHV remains a proper quality measure. The value HV then indicates the achieved hypervolume value regarding to R.

	Steepest Descent		**Hooke & Jeeves**	
φ	Median	Std.Dev.	Median	Std.Dev.
0.0	0.258	0.03357	0.0969	0.03441
0.25	$2.01 \cdot 10^{-3}$	0.00918	$1.32 \cdot 10^{-4}$	$5.3 \cdot 10^{-4}$
0.5	$9.56 \cdot 10^{-5}$	$4.30 \cdot 10^{-5}$	$4.87 \cdot 10^{-5}$	$2.01 \cdot 10^{-6}$
0.75	$6.54 \cdot 10^{-5}$	$1.00 \cdot 10^{-5}$	$4.23 \cdot 10^{-5}$	$2.71 \cdot 10^{-7}$
1.0	$5.26 \cdot 10^{-5}$	$5.99 \cdot 10^{-6}$	$5.26 \cdot 10^{-5}$	$5.99 \cdot 10^{-6}$

Table 1: NDHV values for the relay hybrid SMS-EMOA with Steepest Descent (left) and Hooke & Jeeves (right) on problem ZDT1. Lower values are better.

It seems as if Steepest Descent without SMS-EMOA saves a huge number of fitness evaluations until termination. But a comparison to the achieved fitness after termination, see left part of table 1, shows that the achieved fitness for $\varphi =$

[2]but may terminate earlier

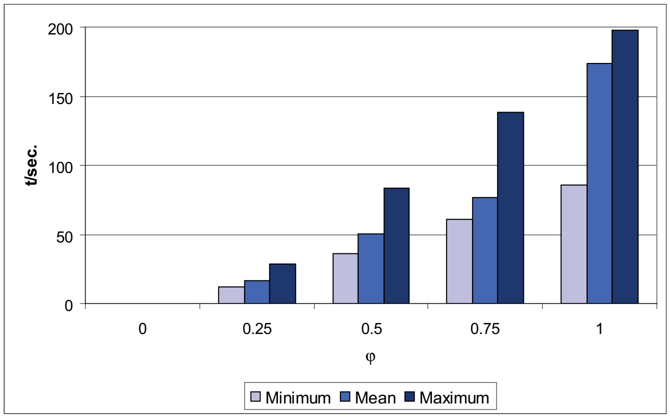

Figure 2: Runtime of the relay hybrid SMS-EMOA with Hooke & Jeeves in seconds for different φ settings. Note that for φ no bars are visible, because the runtime is very close to zero.

0.0 is not satisfying. Only the settings $\varphi = 0.5, 0.75$ and 1.0 deliver a competitive approximation. Nevertheless, about one third of the fitness evaluations can be saved for $\varphi \approx 0.5$ in comparison to the concurrent hybrid and the stand-alone SMS-EMOA.

Furthermore, we analyzed the relay hybrid of the SMS-EMOA with Hooke & Jeeves. Although no considerable improvement in terms of fitness function evaluations or accuracy could have been achieved, we achieved a performance gain – Hooke & Jeeves evaluations are faster than S-Metric evaluations. In each iteration of the SMS-EMOA the hypervolume measure has to be recalculated. Figure 2 shows the runtime of the SMS-EMOA in comparison to the relay hybrid.[3] The corresponding quality results are shown as NDHV values in table 1 on the right. As already observed for the relay hybrid SMS-EMOA with Steepest Descent, the results are only competitive to a φ value of greater than 0.25. However, the speedup of the hybridization can turn out to be an improvement in comparison with the standard SMS-EMOA approach for time-critical applications. The method of Hooke & Jeeves does not perform time consuming computations, so that the overall speedup turns out to be larger, as soon as more function evaluations are spent.

4. CONCURRENT SMS-EMOA HYBRID

Not much effort has been spent on concurrent approaches with the SMS-EMOA in the past. Hence, we decided to concentrate our efforts on this hybridization type. In concurrent approaches, local search is embedded into the evolutionary optimizer.

4.1 Algorithmic Description

Our concurrent hybrid makes use of the multiobjective Newton method by Fliege *et al.* [7] as local search approach. Newton does not need a scalarization of the objective functions or any other mapping, but first and second order derivatives to compute a descent direction. The Newton method can converge in superlinear time to a Pareto-optimum – a great advantage in contrast to other methods like Direct Search and Steepest Descent. Algorithm 1 shows the pseudocode of our concurrent hybridization. A key concept of our approach is the introduction of a probability function $p_{ls}(t)$ for extending the idea presented in Sindhya *et al.* [20] who linearly oscillate the probability for starting local search. We propose a parameterized probability function

$$p_{ls}(t) = \frac{p_{\max} \cdot \Phi(t \bmod (\alpha\mu))}{\Phi(\alpha\mu - 1)} \quad (3)$$

where parameter μ is the population size of the EMOA and $\alpha \in (0, 1]$ is a small constant value − in our experiments later usually chosen as $\alpha = 0.05$. The probability function oscillates with period $\alpha \cdot \mu$ and is linear decreasing in each period. The auxiliary function Φ determines the type of reduction, i.e. linear, quadratic or logarithmic, and has to be defined by the user.

Algorithm 1 Concurrent hybrid EMOA

t:=0
Create random population $\mathcal{P}^t$
Evaluate population $\mathcal{P}^t$
repeat
    Select μ parents out of $\mathcal{P}^t$
    Create population $\mathcal{Q}^t$ with λ offspring
    for $(i := 1$ to $\lambda)$ **do**
        Choose random variable $r \in [0, 1]$.
        if $(r \leq p_{ls}(t))$ **then** local search for $\mathcal{Q}^t[i]$
        end if
    end for
    Evaluate λ offspring
    Create Population $\mathcal{P}^{t+1}$ out of $\mathcal{P}^t$ and $\mathcal{Q}^t$
    t := t+1
until STOP

4.2 Experimental Analysis

The concurrent hybrid SMS-EMOA has been tested empirically. To make this study comparable with other works in the field, we have concentrated on keeping the setup similar to the work of Sindhya *et al.* [20]. One goal of the study is to emphasize the importance of the used probability function p_{ls} that controls the frequency of local search during the optimization process. We tested three different functions using equation 3 and a constant probability p_{ls}. The hybrid variants using equation 3 obtain a value of $\alpha = 0.5$ as proposed by Sindhya *et al.* [20].

1. $p_{ls}(t)$ with $\Phi(x) = x$ (eq. 3)
2. $p_{ls}(t)$ with $\Phi(x) = x^2$ (eq. 3)
3. $p_{ls}(t)$ with $\Phi(x) = log(x)$ (eq. 3)
4. $p_{ls}(t) = 0.01$

Each hybridization with the above probability functions has been run 10 times with fixed random seeds on the ZDT test cases. In each run the hybrid SMS-EMOA is started with a population size of $N = 100$, the SBX recombination operator by Sindhya *et al.* [20] with probability 1.0, and the polynomial mutation operator with probability 1.0. Note, that this setup does not necessarily represent the best choice

[3]The test was performed on a 1.66GHz Intel Core 2 Duo machine with 4GB of memory on Windows Vista.

of parameters for the chosen test functions, but nevertheless allows insights into the hybridization strategies.

Figure 3 shows the development of the Pareto front approximation of the concurrent hybrid SMS-EMOA on the ZDT test cases by Zitzler *et al.* [24]. Again, our analysis is based on the NDHV-measure introduced in paragraph 3.2. Note, that all figures of the fitness development are presented on a logarithmic scale.

4.2.1 ZDT1

The table 2 shows the results of the hybrid SMS-EMOA and the hybrid NSGA-II taken from Sindhya *et al.* [20]. ZDT1 is a convex multiobjective optimization problem. The experimental results are shown in figure 3. The SMS-EMOA hybrids are faster than the stand-alone SMS-EMOA. Surprisingly, the hybrid with a constant probability function p_{ls} shows one of the best approximation abilities. In comparison to NSGA-II and its hybrid variant by Sindhya *et al.* [20] in table 2 the SMS-EMOA variants turn out to be much faster.

ZDT1	Min.	Median	Max.
H-SMS-EMOA (const.)	**0.0001**	**0.0001**	0.0002
H-SMS-EMOA (linear)	**0.0001**	**0.0001**	**0.0001**
H-SMS-EMOA (quadr.)	**0.0001**	**0.0001**	**0.0001**
SMS-EMOA	0.0002	0.0001	0.0072
H-NSGA-II	0.0034	0.0042	0.1630
NSGA-II	0.0043	0.0047	0.0054

Table 2: Comparison of NDHV-Values between hybrid SMS-EMOA and hybrid NSGA-II on ZDT1.

4.2.2 ZDT2

On the non-convex problem ZDT2 the experimental results are even clearer than the results on ZDT1, see figure 3. We observe a clear superiority of the hybrid variants in comparison the stand-alone SMS-EMOA. Again, the constant variant belongs to the best hybridization techniques. Table 3 reveals a clear superiority of the SMS-EMOA hybrids in comparison to the results of NSGA II and its hybrid variants.

ZDT2	Min.	Median	Max.
H-SMS-EMOA (const.)	**0.0001**	**0.0001**	0.0002
H-SMS-EMOA (linear)	**0.0001**	**0.0001**	**0.0001**
H-SMS-EMOA (quadr.)	**0.0001**	**0.0001**	0.0002
SMS-EMOA	0.0005	0.0008	0.0940
H-NSGA-II	0.0037	0.0070	0.0499
NSGA-II	0.0044	0.0053	0.0064

Table 3: Comparison of NDHV-Values between hybrid SMS-EMOA and hybrid NSGA-II on ZDT2.

4.2.3 ZDT3

The fractional Pareto front of ZDT3 is a hard problem for the Newton method. Newton converges into local optima and terminates very fast. Consequently, the exploration behavior is quite poor, but an excellent exploration behavior is necessary for the detection of the various Pareto front fractions. Thus, the hybridizations are not clearly superior, see figure 3. Even worse, the variant with logarithmic probability function fails in comparison to the other variants. The logarithmic variant exhibits the highest probability function density and thus the highest probability for local search. Nevertheless, in comparison to NSGA-II the SMS-EMOA hybrids are still superior, see table 4.

ZDT3	Min.	Median	Max.
H-SMS-EMOA (const.)	**0.0001**	0.0271	0.0271
H-SMS-EMOA (linear)	**0.0001**	**0.0001**	**0.0001**
H-SMS-EMOA (quadr.)	**0.0001**	**0.0001**	0.0271
SMS-EMOA	**0.0001**	0.0002	0.0940
H-NSGA-II	0.0007	0.0009	0.0010
NSGA-II	0.0012	0.0016	0.0023

Table 4: Development of NDHV metrics of the SMS-EAMO and NSGA-II hybrids on problem ZDT3.

4.2.4 ZDT4

ZDT4 is a multiobjective problem with multiple local optima. As stated in the previous paragraph, Newton gets stuck in local optima very fast, thus the exploration abilities are quite poor in multimodal solution space. The experimental results, see figure 3, confirm this assumption. All variants show an equivalent behavior. At least, no deterioration can be observed. In comparison to NSGA-II and its hybrid variants – see table 5 – no consistent picture can be drawn. NSGA-II achieves the best median and maximum fitness, while the H-SMS-EMOA achieves the best fitness at all. We conclude, that Newton's local search is not useful in multimodal fitness landscapes. We left out the experimental analysis on ZDT5 as it is no continuous multiobjective optimization problem.

ZDT4	Min.	Median	Max.
H-SMS-EMOA (const.)	**0.0001**	0.0271	0.0271
H-SMS-EMOA (linear)	0.0202	0.0327	0.0424
H-SMS-EMOA (quadr.)	0.0142	0.0288	0.0345
SMS-EMOA	0.0224	0.0150	0.0472
H-NSGA-II	0.0037	0.0106	0.2230
NSGA-II	0.0042	**0.0047**	**0.0055**

Table 5: Comparison of NDHV-Values between hybrid SMS-EMOA and hybrid NSGA-II on ZDT4.

4.2.5 ZDT6

A clear superiority of SMS-EMOA hybridizations can be observed on the non-convex problem ZDT6, see figure 3 and table 6. Although the approximation behavior of the hybrids resembles the SMS-EMOA's optimization process in the first 3,000 generations, the search of the hybrids accelerates and outperforms the SMS-EMOA clearly. Again, the constant hybrid belongs to the best optimization methods. Further results for the constant probability function are shown in figure 3. The plot shows the outcome of a single run of the concurrent hybrid SMS-EMOA with constant probabilities $p_{ls} = \{0.001, 0.01, 0.1\}$. It can be observed that a high local search probability ($p_{ls} = 0.1$) can have a negative effect on the development of the Pareto front approximation. A too high probability for local search prevents the proper exploitation of the solution space at the beginning of the search and leads to a slower approximation in comparison to $p_{ls} = 0.01$. A same behavior can be observed for $p_{ls} = 0.001$,

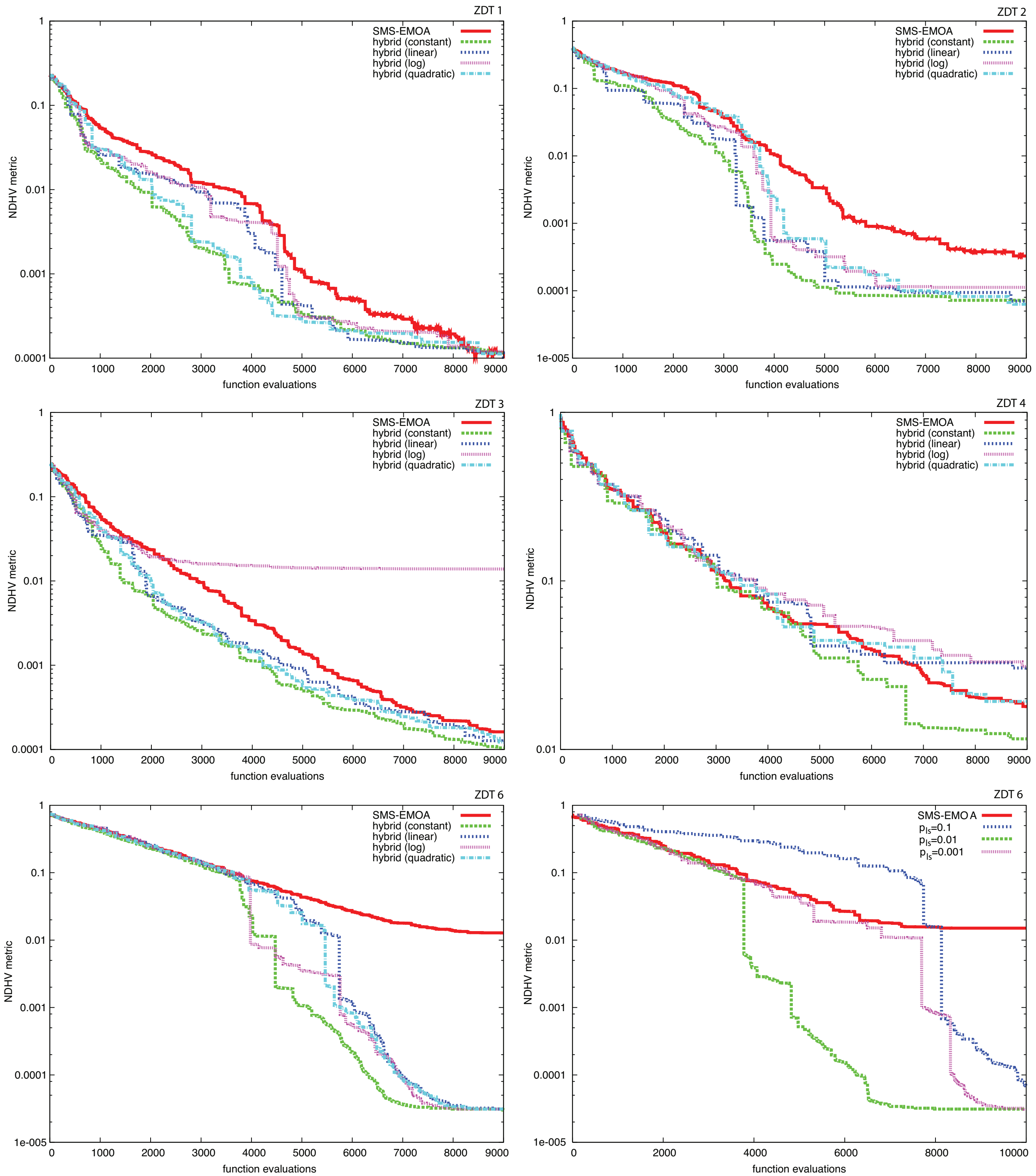

Figure 3: Development of NDHV metric values of SMS-EMOA and the concurrent hybrid on ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6 (from left to right and top to bottom). The bottom right figure shows the results on ZDT6 for concurrent hybrids with constant probability of applying local search.

where the development of the hybrid resembles the stand-alone SMS-EMOA for a long time. But all three local search variants show phases of significant speedup and very fast convergence to the optimum. As no results are available for NSGA-II a comparison is not possible.

ZDT6	**Min.**	**Median**	**Max.**
H-SMS-EMOA (const.)	$3.1 \cdot 10^{-5}$	$3.2 \cdot 10^{-5}$	$3.4 \cdot 10^{-5}$
H-SMS-EMOA (linear)	$3.1 \cdot 10^{-5}$	$3.8 \cdot 10^{-5}$	$3.9 \cdot 10^{-5}$
SMS-EMOA	$8.0 \cdot 10^{-3}$	$1.2 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$

Table 6: A comparison of NDHV-Values on problem ZDT6.

5. CONCLUSIONS

What can we conclude from the experimental results? In general, hybridization seems to make sense as a speedup can be achieved on many problems, in particular on the non-convex ones. With one exception the hybrids were at least as good as the stand-alone optimization methods. Nevertheless, the question for proper hybridizations and parameter settings, e.g. adequate settings for the balance parameter φ, is not easy to answer. For concurrent hybridizations Sudholt [22] has shown that wrong parameterizations can deteriorate the approximation capabilities and that finding a good parameterization can be as difficult as solving the problem.

The introduction of an oscillating probability for starting local search is an intuitive idea: Alternating emphasis on local search and on non-local stochastic search may prevent the optimizer from getting stuck into local optima. But surprisingly, a constant probability for local search shows the most stable approximation capabilities. And the more local optima the solution space exhibits, the worse are the hybridization results: Newton's method gets stuck in local optima and terminates.

The analysis at hand is a first step into the concurrent hybridization of the SMS-EMOA with local search. In the future we will investigate hybridizations with other local search techniques and extend the analysis on further multi-objective problems, also from practice.

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6. REFERENCES

[1] N. Beume, B. Naujoks, and M. Emmerich. SMS-EMOA: Multiobjective Selection based on Dominated hypervolume. *European Journal of Operational Research*, 181(3):1653–1669, 2007.

[2] M. Box, D. Davies, and W. Swann. *Nonlinear Optimization Technique*, volume 5. Oliver and Boyd Ltd., 1969.

[3] K. Deb and T. Goel. A hybrid multi-objective evolutionary approach to engineering shape design. In *Proceedings of the First International Conference on Evolutionary Multicriterion Optimization (EMO 2001)*, pages 385–399, Berlin and Heidelberg, 2001. Springer.

[4] K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A fast and elitist multiobjective genetic algorithm: NSGA-II. *Evolutionary Computation, IEEE Transactions on*, 6(2):182–197, 2002.

[5] M. Ehrgott and X. Gandibleux. Hybrid metaheuristics for multi-objective combinatorial optimization. In F. Almeida, M. J. B. Aguilera, and C. Blum, editors, *Hybrid Metaheuristics*, volume 4030 of *Lecture Notes in Computer Science*, pages 221–259. Springer, 2006.

[6] M. T. M. Emmerich, A. H. Deutz, and N. Beume. Gradient-based/evolutionary relay hybrid for computing pareto front approximations maximizing the s-metric. In *Hybrid Metaheuristics*, pages 140–156, 2007.

[7] J. Fliege, L. M. G. Drummond, and B. Svaiter. Newton's method for multiobjective optimization. *Optimization Online*, 2008.

[8] J. Fliege and B. Svaiter. Steepest descent methods for multicriteria optimization. *Mathematical Methods of Operations Research*, 51(3):479–494, 2000.

[9] R. Hooke and T. Jeeves. Direct search: Solution of numerical and statistical problems. *Journal of the ACM*, 8(2):212–229, 1961.

[10] H. Ishibuchi and T. Murata. Multi-objective genetic local search algorithm. In *International Conference on Evolutionary Computation*, pages 119–124, 1996.

[11] H. Ishibuchi and K. Narukawa. Some issues on the implementation of local search in evolutionary multiobjective optimization. In *Genetic and Evolutionary Computation - GECCO 2004*, pages 1246–1258, 2004.

[12] H. Ishibuchi and T. Yoshida. Implementation of local search in hybrid multi-objective genetic algorithms: A case study on flowshop scheduling. In *Proc. of fourth Asia-Pacific Conference on Simulated Evolution And Learning*, pages 193–197, 2002.

[13] H. Ishibuchi, T. Yoshida, and T. Murata. Balance between genetic search and local search in memetic algorithms for multiobjective permutation flowshop scheduling. *IEEE Trans. on Evolutionary Computation*, 7:204–223, 2003.

[14] A. Jaszkiewicz. Genetic local search for multiple objective combinatorial optimization. Technical report, 1998.

[15] S. Z. Martínez and C. A. C. Coello. A proposal to hybridize multi-objective evolutionary algorithms with non-gradient mathematical programming techniques. In G. Rudolph, T. Jansen, S. M. Lucas, C. Poloni, and N. Beume, editors, *PPSN*, volume 5199 of *Lecture Notes in Computer Science*, pages 837–846. Springer, 2008.

[16] G. R. Raidl. A unified view on hybrid metaheuristics. In F. Almeida, M. J. B. Aguilera, and C. Blum, editors, *Hybrid Metaheuristics*, volume 4030 of *Lecture Notes in Computer Science*, pages 1–12. Springer, 2006.

[17] O. Schuetze, G. Sanchez, and C. A. C. Coello. A new memetic strategy for the numerical treatment of multi-objective optimization problems. In *GECCO '08: Proceedings of the 10th annual conference on*

Genetic and evolutionary computation, pages 705–712, New York, NY, USA, 2008. ACM.

[18] H. Schwefel. John Wiley & Sons, Inc., 1995.

[19] P. K. Shukla. Gradient based stochastic mutation operators in evolutionary multi-objective optimization. In *Adaptive and Natural Computing Algorithms*, pages 58–66, 2007.

[20] K. Sindhya, K. Deb, and K. Miettinen. A local search based evolutionary multi-objective optimization approach for fast and accurate convergence. In G. Rudolph, T. Jansen, S. M. Lucas, C. Poloni, and N. Beume, editors, *Parallel Problem Solving from Nature - PPSN X*, 2008.

[21] R. Steuer. *Multiple Criteria Optimization: Theory, Computation and Application*. John Wiley & Sons, New York, NY, 1985.

[22] D. Sudholt. *Computational Complexity of Evolutionary Algorithms, Hybridizations, and Swarm Intelligence*. PhD thesis, Technische Universtät Dortmund, Dortmund, 2008.

[23] E.-G. Talbi. A taxonomy of hybrid metaheuristics. *Journal of Heuristics*, 8(5):541–564, 2002.

[24] E. Zitzler, K. Deb, and L. Thiele. Comparison of multiobjective evolutionary algorithms: Empirical results. *Evolutionary Computation*, 8:173–195, 2000.