Computational Intelligence
Winter Term 2011/12

Prof. Dr. Günter Rudolph
Lehrstuhl für Algorithm Engineering (LS 11)
Fakultät für Informatik
TU Dortmund

Plan for Today

- Organization (Lectures / Tutorials)
- Overview CI
- Introduction to ANN
  - McCulloch Pitts Neuron (MCP)
  - Minsky / Papert Perceptron (MPP)

Organizational Issues

Who are you?
either
studying “Automation and Robotics” (Master of Science)
Module “Optimization”
or
studying “Informatik”
- BA-Modul “Einführung in die Computational Intelligence”
- Hauptdiplom-Wahlvorlesung (SPG 6 & 7)

Who am I?

Günter Rudolph
Fakultät für Informatik, LS 11
Guenter.Rudolph@tu-dortmund.de ← best way to contact me
OH-14, R. 232 ← if you want to see me

office hours:
Tuesday, 10:30–11:30am
and by appointment
Organizational Issues

Lectures
Wednesday 10:15-11:45 OH-14, R. 304

Tutorials
Wednesday 12:15-13:45 OH-14, R. 304, bi-weekly
or 16:15-17:45 OH-14, R. 304, bi-weekly

Tutor
Dipl.-Inform. Nicola Beume, LS 11

Information

Slides
see web

Literature
see web

---

Prerequisites

Knowledge about
• mathematics,
• programming,
• logic
is helpful.

But what if something is unknown to me?
• covered in the lecture
• pointers to literature

... and don’t hesitate to ask!

---

Overview “Computational Intelligence“

What is CI?
⇒ umbrella term for computational methods inspired by nature

• artificial neural networks
• evolutionary algorithms
• fuzzy systems
• swarm intelligence
• artificial immune systems
• growth processes in trees
• ...

backbone
new developments

---

Overview “Computational Intelligence“

• term „computational intelligence“ coined by John Bezdek (FL, USA)
• originally intended as a demarcation line
⇒ establish border between artificial and computational intelligence
• nowadays: blurring border

our goals:
1. know what CI methods are good for!
2. know when refrain from CI methods!
3. know why they work at all!
4. know how to apply and adjust CI methods to your problem!
Biological Prototype

- Neuron
  - Information gathering (D)
  - Information processing (C)
  - Information propagation (A / S)

human being: $10^{12}$ neurons
electricity in mV range
speed: 120 m / s

Abstraction

function $f$:
$$f(x_1, x_2, \ldots, x_n)$$

McCulloch-Pitts-Neuron 1943:
$$x_i \in \{0, 1\} =: B$$
$$f: \mathbb{B}^n \rightarrow \mathbb{B}$$

1943: Warren McCulloch / Walter Pitts

- description of neurological networks
  → modell: McCulloch-Pitts-Neuron (MCP)

- basic idea:
  - neuron is either active or inactive
  - skills result from connecting neurons

- considered static networks
  (i.e. connections had been constructed and not learnt)
McCulloch-Pitts-Neuron

n binary input signals $x_1, \ldots, x_n$

threshold $\theta > 0$

$$f(x_1, \ldots, x_n) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} x_i \geq \theta \\ 0 & \text{else} \end{cases}$$

boolean OR

$$\Rightarrow \text{can be realized: } \geq 1$$

$$\theta = 1$$

boolean AND

$$\Rightarrow \text{can be realized: } \geq n$$

$$\theta = n$$

in addition: $m$ binary inhibitory signals $y_1, \ldots, y_m$

- if at least one $y_j = 1$, then output = 0
- otherwise:
  - sum of inputs \(\geq\) threshold, then output = 1
  - else output = 0

Example:

$$F(x) = x_1x_2\bar{x}_3 \lor \bar{x}_1\bar{x}_2\bar{x}_3 \lor x_1\bar{x}_4$$

Theorem:

Every logical function $F: \mathbb{B}^n \rightarrow \mathbb{B}$ can be simulated with a two-layered McCulloch/Pitts net.

Proof: (by construction)

1. Every clause gets a decoding neuron with $\theta = n$
   \(\Rightarrow\) output = 1 only if clause satisfied (AND gate)

2. All outputs of decoding neurons
   are inputs of a neuron with $\theta = 1$ (OR gate)

q.e.d.
Generalization: inputs with weights

\[ 0.2 x_1 + 0.4 x_2 + 0.3 x_3 \geq 0.7 \]

\[ 2 x_1 + 4 x_2 + 3 x_3 \geq 7 \]

\[ \Rightarrow \text{equivalent!} \]

Theorem:
Weighted and unweighted MCP-nets are equivalent for weights \( \mathbb{Q}^+ \).

Proof:
\[ \Rightarrow^* \]
Let \( \sum_{i=1}^{n} \frac{a_i}{b_i} x_i \geq \frac{a_0}{b_0} \quad \text{with } a_i, b_i \in \mathbb{N} \)
\[
\Rightarrow^* \quad \text{Multiply with } \prod_{i=0}^{n} b_i \quad \text{yields inequality with coefficients in } \mathbb{N}
\]
Duplicate input \( x_i \), such that we get \( a_1 b_1 \cdots b_{i-1} b_{i+1} \cdots b_n \) inputs.
Threshold \( \theta = a_0 b_1 \cdots b_n \)
\[ \Leftarrow^* \]
Set all weights to 1. q.e.d.

Introduction to Artificial Neural Networks

Conclusion for MCP nets

+ feed-forward: able to compute any Boolean function
+ recursive: able to simulate DFA
- very similar to conventional logical circuits
- difficult to construct
- no good learning algorithm available

Perceptron (Rosenblatt 1958)
\[ \Rightarrow \text{complex model } \Rightarrow \text{reduced by Minsky & Papert to what is } \Rightarrow \text{Minsky-Papert perceptron (MPP), 1969 } \Rightarrow \text{essential difference: } x \in [0,1] \subset \mathbb{R} \]

What can a single MPP do?

\[ w_1 x_1 + w_2 x_2 \geq \theta \quad \text{with } \theta \quad \text{isolation of } x_2 \text{ yields:} \]
\[ x_2 \geq \frac{\theta - w_1 x_1}{w_2} \]

Example:
\[ 0.9 x_1 + 0.8 x_2 \geq 0.6 \]
\[ \Leftrightarrow x_2 \geq \frac{3 - 9}{4 - 8} x_1 \]
\[ \Rightarrow x_2 \geq \frac{3}{4} - \frac{9}{8} x_1 \]
\[ \text{separating line separates } \mathbb{R}^2 \text{ in 2 classes} \]
how to leave the „dead end“:

1. **Multilayer Perceptrons:**

   \[ g(x_1, x_2) = 2x_1 + 2x_2 - 4x_1x_2 - 1 \]
   \[ \Rightarrow 0 < \theta \]
   \[ \Rightarrow w_2 \geq \theta \]
   \[ \Rightarrow w_1 \geq \theta \]
   \[ \Rightarrow w_1 + w_2 \geq 2\theta \]
   \[ \Rightarrow w_1 + w_2 < \theta \]
   \[ \text{contradiction!} \]

2. **Nonlinear separating functions:**

   XOR
   \[ g(x_1, x_2) = 2x_1 + 2x_2 - 4x_1x_2 - 1 \]
   \[ \Rightarrow 0 < \theta \]
   \[ \Rightarrow w_2 \geq \theta \]
   \[ \Rightarrow w_1 \geq \theta \]
   \[ \Rightarrow w_1 + w_2 \geq 2\theta \]
   \[ \Rightarrow w_1 + w_2 < \theta \]
   \[ \text{contradiction!} \]

How to obtain weights \( w_i \) and threshold \( \theta \)?

- **1969: Marvin Minsky / Seymour Papert**
  - book *Perceptrons* → analysis math. properties of perceptrons
  - disillusioning result: perceptions fail to solve a number of trivial problems!
    - XOR-Problem
    - Parity-Problem
    - Connectivity-Problem
  - „conclusion“: All artificial neurons have this kind of weakness!
    \( \Rightarrow \) research in this field is a scientific dead end!
  - consequence: research funding for ANN cut down extremely (~ 15 years)

- **How to obtain weights \( w_i \) and threshold \( \theta \)?**
  - as yet: by construction
  - example: NAND-gate
    \[ g(x_1, x_2) = 2x_1 + 2x_2 - 4x_1x_2 - 1 \]
    \[ \Rightarrow 0 \geq \theta \]
    \[ \Rightarrow w_2 \geq \theta \]
    \[ \Rightarrow w_1 \geq \theta \]
    \[ \Rightarrow w_1 + w_2 < \theta \]
    \[ \text{requires solution of a system of linear inequalities (\( \subset P \))} \]
    \[ (\text{e.g.: } w_1 = w_2 = -2, \ \theta = -3) \]
  - now: by „learning“ / training
Perceptron Learning

Assumption: test examples with correct I/O behavior available

Principle:
1. choose initial weights in arbitrary manner
2. feed in test pattern
3. if output of perceptron wrong, then change weights
4. goto (2) until correct output for all test patterns

graphically:

→ translation and rotation of separating lines

Introduction to Artificial Neural Networks

Example

\[
P = \left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} \right\}
\]

\[
N = \left\{ \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} \right\}
\]

threshold as a weight: \( w = (\theta, w_1, w_2)' \)

\[
x_1 \\
w_1 \\
\frac{x_2}{w_2} \geq 0
\]

suppose initial vector of weights is \( w^{(0)} = (1, -1, 1)' \)

P: set of positive examples
N: set of negative examples

1. choose \( w_0 \) at random, \( t = 0 \)
2. choose arbitrary \( x \in P \cup N \)
3. if \( x \in P \) and \( w^t x > 0 \) then goto 2
   if \( x \in N \) and \( w^t x \leq 0 \) then goto 2
4. if \( x \in P \) and \( w^t x \leq 0 \) then \( w^{t+1} = w_t + x; t++ \) goto 2
5. if \( x \in N \) and \( w^t x > 0 \) then \( w^{t+1} = w_t - x; t++ \) goto 2
6. stop? If I/O correct for all examples!

remark: algorithm converges, is finite, worst case: exponential runtime

We know what a single MPP can do.
What can be achieved with many MPPs?

Single MPP \( \Rightarrow \) separates plane in two half planes
Many MPPs in 2 layers \( \Rightarrow \) can identify convex sets

\[
A \\
B
\]

\( \forall a,b \in X: \lambda a + (1-\lambda) b \in X \)
\( \lambda \in (0,1) \)
Introduction to Artificial Neural Networks

Single MPP ⇒ separates plane in two half planes
Many MPPs in 2 layers ⇒ can identify convex sets
Many MPPs in 3 layers ⇒ can identify arbitrary sets
Many MPPs in > 3 layers ⇒ not really necessary!

arbitrary sets:
1. partitioning of nonconvex set in several convex sets
2. two-layered subnet for each convex set
3. feed outputs of two-layered subnets in OR gate (third layer)