Computational Intelligence

Winter Term 2011/12

Prof. Dr. Günter Rudolph Lehrstuhl für Algorithm Engineering (LS 11)

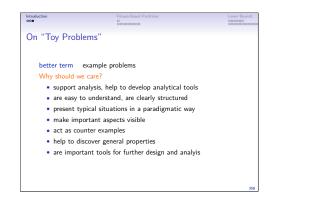
Fakultät für Informatik

TU Dortmund

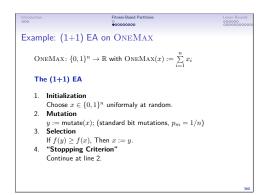
Introduction eoo	Fitness-Based Partitions Lower I 000000000000000000000000000000000000	
Evoluti	onary Algorithms	
We k	now	
•	what evolutionary algorithms are and	
•	how we can design evolutionary algorithms.	
What	t do we want to do now?	
What	t do we do if we design a problem-specific algorithm?	
0	prove its correctness	
2	analyze its performance: (expected) run time	
	t does this mean for evolutionary algorithms in the context of nization?	
	prove that max. $f\mbox{-value}$ in population converges to global max. of f for $t\to\infty$	
	analyze how long this takes on average: expected optimization time	
		356

Introduction 000	Fitness-Based Partitions 000000000	Lower Bounds 000000 00000000000
Plans for Toda	/	
1 Introduction Motivation	1	
2 Fitness-Based Method of Applicatio	Fitness-Based Partitions	
3 Lower Bound Direct Lov Drift Anal	ver Bounds	

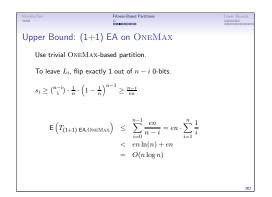
Introduction 000		Fitness-Based Partitions 0 000000000	Lower Bounds
Analysis o	f Evolu	tionary Algorithms	
		lutionary algorithms do we want to analyze?	
clearly	all kinds	of evolutionary algorithms	
more rea		ery simple evolutionary algorithms t least as starting point	
For wha	t kind of	problems do we want to do analysis?	
clearly	all kinds	of problems	
more rea		ery simple problems — "toy problems" t least as starting point	
			357



Introduction 000	Fitness-Based Partitions	Lower Bounds 000000 00000000000
	with f -based partitions based partitions works well with plus-s	election.
Definition		
	$\rightarrow \mathbb{R}$. A partition L_0, L_1, \dots, L_k of { on iff the following holds.	$\{0,1\}^n$ is called
	$(\dots, k): \forall x \in L_i: \forall y \in L_j: (i < j = {\mathbb{E}} \{0, 1\}^n \mid f(x) = \max \{f(y) \mid y \in \{0, 1\}^n \}$	
Often the trivia	al <i>f</i> -based parition works well.	
TOP C 7 1	$x \in \{0,1\}^n\} -1$ $\{,1\}^n\} = \{f_0,f_1,\ldots,f_k\}$ with $f_0 < f_2$	$f_1 < \dots < f_k$
for $i \in \{0, 1, \ldots$., k}: $L_i := \{x \in \{0, 1\}^n \mid f(x) = f_i$;}



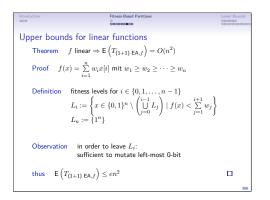
Introduction	Fitness-Based Partitions	Lower Bounds
000	00000000	0000000
Method: <i>f</i> -ba	ased partitions	
Key Observ	ation:	
(1+1) EA lea	aves each fitness layer at most once.	
	on the probability to leave L_i :	
$s_i := \min_{x \in L_i} \sum_{j=1}^{j}$	$\sum_{i+1}^k \sum_{y \in L_j} p_m^{H(x,y)} \cdot (1-p_m)^{n-H(x,y)}$	
Upper bound	I on the expected time needed to leave L_i	:
E (time to le	ave $L_i) \leq 1/s_i$	
	d on the expected optimization time:	
$E\left(T_{(1+1) EA}\right)$	$f \leq \sum_{i=0}^{k-1} 1/s_i$	
		361

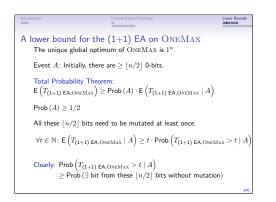


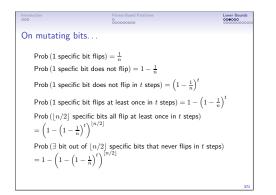
Introduction	Fitness-Based Partitions	Lower Bounds
000	00000000	000000000000000000000000000000000000000
Linear Funct	ions	
Observation	$\begin{aligned} \text{OneMax}(x) &= \sum_{i=1}^n x[i] \\ \text{is of the form } f(x) &= w_0 + \sum_{i=1}^n w_i \cdot x[i] \end{aligned}$	
	<i>i</i> =1	
Definition	$f \colon \{0,1\}^n \to \mathbb{R}$ is called linear if f is of the form $f(x) = w_0 + \sum_{i=1}^n w_i \cdot x[i]$	
Are all linea	r functions like ONEMAX?	
Definition	different extreme example BINVAL: $\{0,1\}^n \to \mathbb{R}$ with BINVAL $(x) = \sum_{i=1}^n 2^{n-i} \cdot x[i]$	
		363

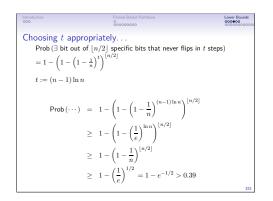
Introduction 000	Fitness-Based Partitions	Lower Bounds 000000
Upper bou	nd for $E\left(T_{(1+1) \;EA,\mathrm{BinVAL}} ight)$	00000000000
Consider	trivial fitness levels $\forall i \in \{0,1,\ldots,2^n-1\}: L_i := \{x \in \{0,1\}^n \mid z \in \{0,1\}^n \mid z \in \{0,1\}^n \mid z \in \{0,1\}^n \}$	BinVal(x) = i
without c	considering s_i at best upper bound $\geq 2^n - 1$ as	chievable
Observati	ion for good upper bounds number of fitness lo needs to be small	evels
$\forall i$	$ \begin{split} & \text{for elever fitness levels} \\ & \in \{0, 1, \dots, n-1\} ; \\ & := \left\{ x \in \{0, 1\}^n \setminus \begin{pmatrix} i - 1 \\ \bigcup_{j=0}^{i-1} L_j \end{pmatrix} \mid \text{BinVal}(x) < \sum_{j=0}^i \end{pmatrix} \end{split} $	2^{n-1-j}
		364

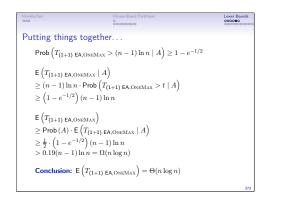
Introduction 000	Fitness-Based Partitions	Lower Bounds
	00000000	000000000
Upper bound for	or $E\left(T_{(1+1) EA, \operatorname{BinVal}}\right)$ (II)	
$\forall i \in \{0, 1, \dots, $	n-1:	
$L_i := \begin{cases} x \in \{0\} \end{cases}$	$(1)^n \setminus \left(\bigcup_{j=0}^{i-1} L_j\right) BINVAL(x) < \sum_{j=0}^{i} 2$	n-1-j
		J
obvious $s_i \ge$	$\frac{1}{n}\left(1-\frac{1}{n}\right)^{n-1} \ge \frac{1}{en}$	
Theorem E ($T_{(1+1) EA,BINVAL} \le en^2$	
,		







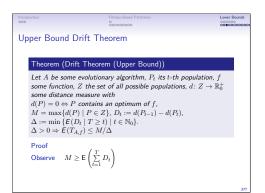




Introduction 000	Fitness-Based Partitions 000000000	Lower Bounds
The Coupo	on Collector Theorem	
Scenario	Collect coupons of n different types unless you have at least one of each type. Obtain single coupons, each time independently each type with equal probability. Let T be the number of coupons obtained at the	end.
Theorem		
- ()	$) = n \ln n + O(n)$	
$2 \forall \beta \ge$	1: Prob $(T > \beta n \ln n) \le n^{-(\beta-1)}$ \mathbb{R} : Prob $(T > n \ln n + cn) \le 1 - e^{-e^{-c}}$	

Introduction 000	Fitness-Based Partitions 0 000000000	Lower Bounds
A More Flex	ibel Proof Method	
Observation	S	
 f-base 	d partitions restricted to "well behaving"	functions
 direct 	ower bound often too difficult	
How can we	e find a more flexibel method?	
Observation	f-based partition measure progress b	$y f(x_{t+1}) - f(x_t)$
Idea cons	ider a more general measure of progress	
	stance $d: Z \to \mathbb{R}^+_0$, (Z set of all populat ith $d(P) = 0 \Leftrightarrow P$ contains optimal solution	
Caution	'Distance" need not be a metric!	
		375

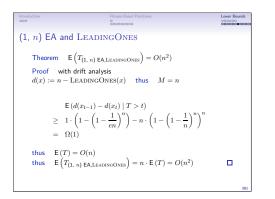
Introduction 000	0 0	wer Bounds
Drift		
Define	distance $d: Z \to \mathbb{R}^+_0$, (Z set of all populations) with $d(P) = 0 \Leftrightarrow P$ contains optimal solution	
Observat	ion $T = \min\{t \mid d(P_t) = 0\}$	
Consider	$\label{eq:maximum distance } \begin{array}{l} \mbox{maximum distance } M := \max{\{d(P) \mid P \in Z\}}, \\ \mbox{decrease in distance } D_t := d(P_{t-1}) - d(P_t) \end{array}$	
Definitio	n $E(D_t \mid T \ge t)$ is called drift.	
	tic point of view $\Delta := \min \{ E \left(D_t \mid T \ge t \right) \mid t \in \mathbb{N}_0 \}$	
Drift The	eorem (Upper Bound) $\Delta > 0 \Rightarrow E(T) \le M/\Delta$	
		376



Introduction 000	0	Lower Bounds
$M \ge$	Drift Theorem (Upper Bound) $E\left(\sum_{t=1}^{T} D_{t}\right) = \sum_{t=1}^{\infty} \operatorname{Prob}\left(T = t\right) \cdot E\left(\sum_{i=1}^{T} D_{i} \mid T = \sum_{t=1}^{\infty} \operatorname{Prob}\left(T = t\right) \cdot \sum_{i=1}^{t} E\left(D_{i} \mid T = t\right)$ $\sum_{t=1}^{\infty} \sum_{i=1}^{t} \operatorname{Prob}\left(T = t\right) \cdot E\left(D_{i} \mid T = t\right)$	
=	$\sum_{i=1}^{\infty}\sum_{t=i}^{\infty}Prob\left(T=t\right)\cdotE\left(D_{i}\mid T=t\right)$	378

Introduction 000	Fitness-Based Partitions	Lower Bounds
Proof	of the Drift Theorem (Upper Bound) (cont.)	
=	$\begin{split} &\sum_{i=1}^{\infty}\sum_{t=i}^{\infty}Prob\left(T=t\right)\cdotE\left(D_{i}\mid T=t\right) \\ &\sum_{i=1}^{\infty}\sum_{t=i}^{\infty}Prob\left(T\geq i\right)\cdotProb\left(T=t\mid T\geq i\right)\cdotE\left(D_{i}\mid T=t\right) \\ &\sum_{i=1}^{\infty}Prob\left(T\geq i\right)\sum_{t=i}^{\infty}Prob\left(T=t\mid T\geq i\right)\cdotE\left(D_{i}\mid T=t\land T_{i}) \\ &\sum_{i=1}^{\infty}Prob\left(T\geq i\right)\sum_{t=i}^{\infty}Prob\left(T=t\mid T\geq i\right)\cdotE\left(D_{i}\mid T=t\land T_{i}) \\ &\sum_{i=1}^{\infty}Prob\left(T\geq i\right)E\left(D_{i}\mid T\geq i\right)\geq\Delta\cdot\sum_{i=1}^{\infty}Prob\left(T\geq i\right) = \\ &s E\left(T\right)\leq\frac{M}{\Delta} \end{split}$	$T \ge i$) $T \ge i$)
		379

Introduction 000	Fitness-Based Partitions Lower O 00000 000000000 00000	
LEADINGO	NES Using the Drift Theorem	
Remember	$E\left(T_{(1+1)\ EA,\mathrm{LEADINGONES}}\right) = O(n^2)$ using $f\text{-based partitions}$	
	$\begin{aligned} d(x) &:= n - \text{LEADINGONES}(x) \\ M &= \max \left\{ d(x) \mid x \in \{0, 1\}^n \right\} = n \end{aligned}$	
	$\begin{split} & \mathbb{E}\left(d(x_{t-1}) - d(x_t) \mid T > t\right) \geq 1 \cdot \frac{1}{n} \cdot \left(1 - \frac{1}{n}\right)^{n-1} \geq \frac{1}{en} \\ & T \right) \leq \frac{n}{1/en} = en^2 \end{split}$	
same result	Is there no advantage?	
Advantage	being more general and applicable	
	f-based partitions not applicable for comma selection	
		380



Introduction 000	Fitness-Based Partitions	Lower Bounds
Another Drif	t Theorem	
Remember	$\begin{array}{l} \text{distance } d\colon Z\to \mathbb{R}^+_0 \text{ with } d(P)=0 \Leftrightarrow P \text{ optim:}\\ M:=\max\left\{d(P)\mid P\in Z\right\}, D_t:=d(P_{t-1})-d(P_{t-1})-d(P_{t-1})-d(P_{t-1})-d(P_{t-1})-d(P_{t-1})\right\}\\ \Delta:=\min\left\{E\left(D_t\mid T\geq t\right)\mid t\in\mathbb{N}_0\right\}\\ \Delta>0\RightarrowE(T)\leq \frac{M}{\Delta} \end{array}$	
Observe .	M can be replaced by $E\left(d(P_0) ight)$	
In addition		
	Let $d: Z \to \mathbb{N}_0$ be distance, rest as before. $\exists c \in \mathbb{R}^+ : \forall P_{t-1} : E \left(d(P_{t-1}) - d(P_t) \mid P_t \right) \ge \frac{d(P_{t-1})}{c}$ $\Rightarrow E \left(T \right) \le c \cdot E \left(H_{d(P_0)} \right)$	1)
Proof ide	a Apply drift theorem to $d' := H_d$.	

